Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series

Integral Calculus – Introduction

Differential Calculus is about rates of change.

Integral Calculus is about total change.

Basic Issue:

Given the rate of change of a function, **find the total change** of this function between two points.

We are given the rate of change of $F(t)$ and want to know how much F changes from $t = a$ to $t = b$.

We start with some water flowing through a pipe and can measure the flow rate.

We want to know the amount of water that comes out the end from $t = a$ to $t = b$.

Importance:

Models that are interested in one quantity are often built from assumptions about its rate of change. This leads to models built from differential equations. To solve these we must "integrate".

Second version:

Given a function $g(t)$ we want to find another function $F(t)$ such that

$$\frac{dF(t)}{dt} = g(t)$$

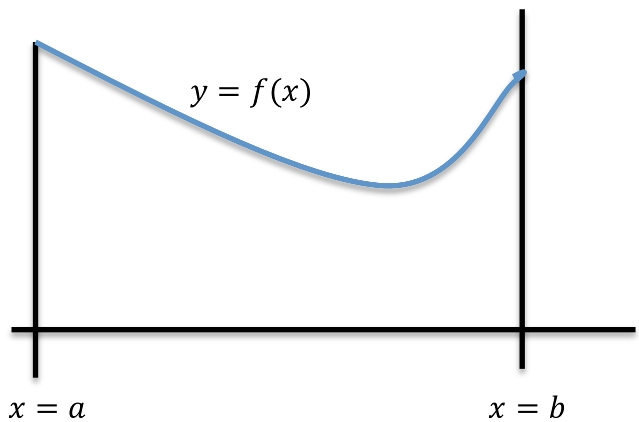
This $F(t)$ is called an **anti-derivative** of $g(t)$.

The anti-derivative measures the total change.

So Integral Calculus turns into a hunt for anti-derivatives.

Third Version:

Graph $y = f(x)$ and consider the area under the curve.



As b increases the rate of change of the area is the height of the right hand side.

The rate of change of the area under the graph of $y = f(x)$ is the function $f(x)$.

So the area under the graph of $y = f(x)$ is an anti-derivative of the function $f(x)$.

Summary

1. The integral of a rate of change is the total change.

2. The integral can be calculated with an anti-derivative.

3. The integral of a function is the area under the graph of the function